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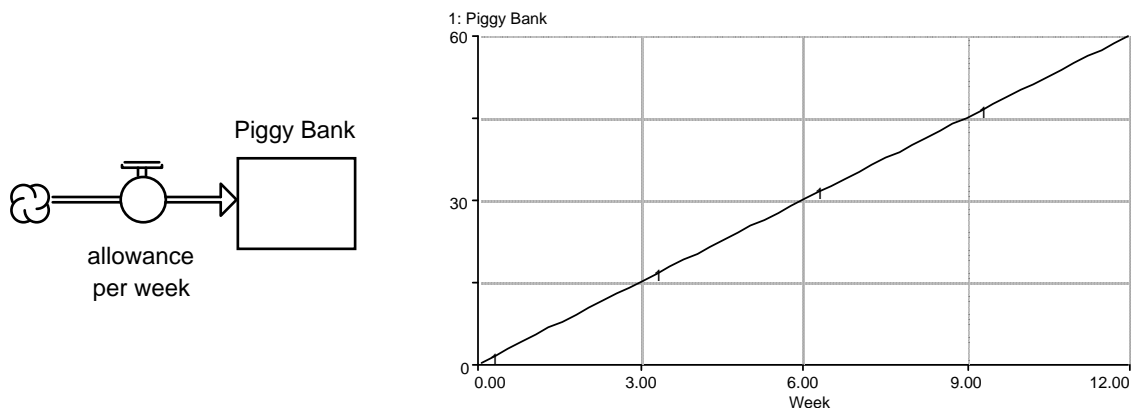
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Generic Processes I: Models Producing Linear Behavior¹

Generic models usually have fairly simple diagram structures producing a common behavior pattern that is observed in many different situations. By examining the behavior pattern of a specific system and remembering the behavior pattern shown by different types of generic models, we can find clues about the type of model structure needed to represent some new system in which we are interested.

In this first simple model structure there is a stock and a flow, pointing toward or away from the stock. The most important characteristic of this structure type is that the stock value either increases or decreases by a constant value each time period. A typical model of this type is shown:

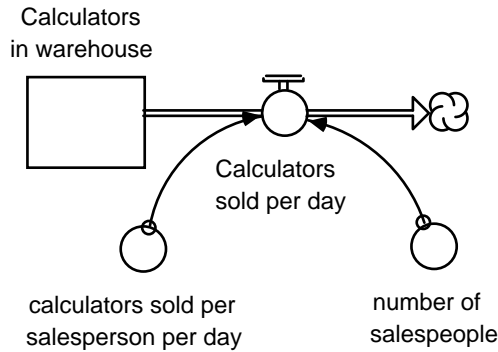


1. The name of this type of model is derived from the shape of the graph. Explain why this generic model produces linear behavior.

In order for a model to be linear, the *rate of change* of the quantity in the stock must be constant. The stock can be increasing or decreasing over time, but it must be changing at a steady and unvarying pace.

¹ This lesson was written by Eileen Rogers.

2. An example of a linear decreasing model could be inventory in a warehouse that ships the same number of items each day. We will assume that this warehouse is going out of business, so no new calculators are being sent to the warehouse.



2(a). Notice that this model has additional components shown as converters. Based on the titles of the converters, write the equation that must have been used to calculate the flow value in this model. Be sure to include appropriate units.

2(b). Build this model. Set the initial number of calculators to 10,000 {calculators}. Assume there are 20 {salespersons} members of the sales staff and that they each sell 15 {calculators per salesperson per day}. Set the lower limit of the graph to 0 and the upper limit to 20,000. Use Run Specs to set the simulation time to 30 days. Make a graph showing the value of the calculators in the warehouse over time.

2(c). How many calculators were left in inventory on the 11th day?

2(d). Now, change your model slightly. A recession has hit and the salespeople can only sell five calculators each day. (Assume for the time being that nobody gets laid off.) Keeping everything else the same (that is, start with 10,000 calculators in the warehouse and 20 salespersons), run the model again and look at the new graph.

How does this new graph differ from the graph produced in 2(b) above? Explain why this difference exists.

2(e). Change the scenario one more time. This time, consider a factory that starts with 20,000 calculators and a sales staff of 20 who each sell 15 calculators a day. What is the major difference between this graph and your first graph?

2(f). Which part of a linear STELLA model determines the “steepness” of the graph of the stock value?

2(g). Which part of the STELLA model determines where the graph will touch the vertical axis?

A Study of Population Growth¹

Introduction

System Dynamics provides us with a very powerful tool for studying the behavior of complex systems over time. Once a model has been constructed, simulations can be run and model values can be recorded on graphs and tables. The graphs are usually plotted with time on the horizontal axis, and are therefore known as *behavior-over-time graphs* (BOTGs).

Reference Behavior

You may be asked to sketch “reference” behavior graphs before developing a model and running the simulations. A reference graph is usually a BOTG. You should sketch the reference graph on paper, describing the behavior you are trying to model. This can often be quite difficult when you don’t understand the problem very well, but it is crucial that you make some attempt to describe the simplest possible behavior, and then develop a very simple model to simulate this behavior. Even if your guess is wrong it will focus your attention on the important behavior of the system, and make you think much more critically about it.

Feedback Loops

Another critical component of System Dynamics is the consideration and study of “feedback loops” within the complex system. We are typically taught to think about “cause and effect,” but in reality, the “effect” often loops back to influence its own cause. These relationships are very hard to estimate without a computer simulation, especially when they occur over long periods of time or over distances in space. Examples might include changes in interest rates by the federal government as a result of excessive spending by the general population or as a result of inflation. These situations may be “caused” by the behavior of the general population, but the “effect” (higher interest rates) may not happen until months later. The decision will be made in Washington, D.C. and reported in the paper, but most people will never realize that their actions led to this result. To make matters more complicated, people may begin to buy more because of *rumors* that interest rates will be going up, so they better buy now before prices increase. Their actions therefore act to aggravate the situation and eventually “cause” interest rates to go up!

¹ This lesson was written by Diana Fisher and Ed Gallaher.

The First Population Model

This tutorial will concentrate on a study of population because it is a model whose elements are relatively easy to understand. We start with an overview of the important factors in this model. In a population model the focus of study is to observe changes in the number of people in the group at any point in time. Consequently, we will have a special icon in our model identified as population.

We all know there are many factors that affect the change in the number of people in a population. In order to keep this model simple we will limit our study to only two flows – one for *births* and one for *deaths*.

Using Your Journal



Draw three different possible population graphs (no numbers) that span 100 years. Label the horizontal axis *years* and the vertical axis *population*. Give three, one sentence explanations (one for each graph) of what might cause each type of pattern.

Part I: The Birth Flow and Reinforcing Feedback Loop

Consider the births. Since births only increase a population we will design part of our model to include this fact. To keep the model very simple, let us assume that each year the population will be increased by a certain number of babies and that the birth rate (or birth fraction) value will not change over the entire simulation time for this model. As a convenient (and reasonable for the US) value, let's assume that there are 15 births for every 1000 people in the population each year. We will identify that fraction as the birth fraction.

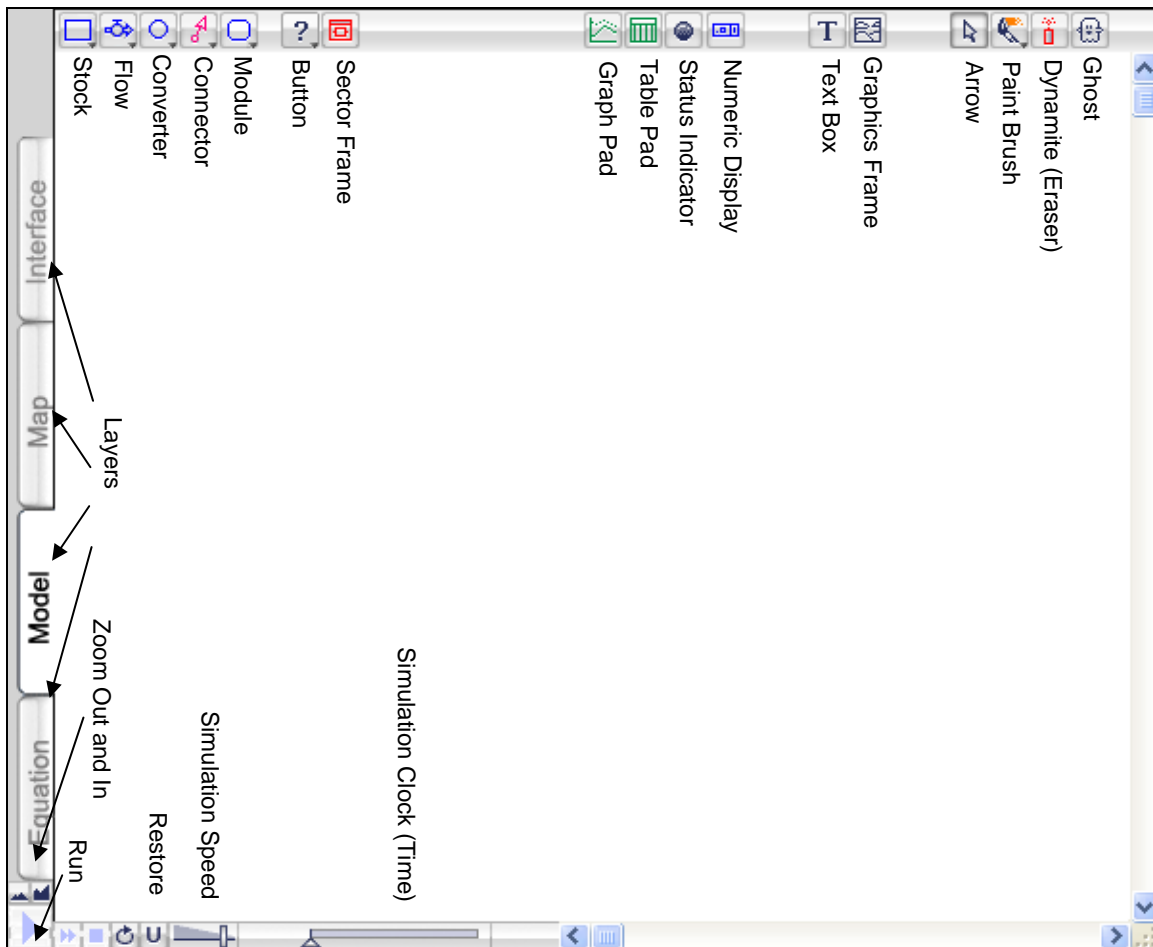
Births add people to a population, which, because the population is now increased, causes even more births to occur. A loop connection is created between the number of births and the population. This loop would be identified as a "reinforcing" feedback loop since it will lead to an ever-increasing number of people in the population.

Let's start the population at 100,000 persons, as we begin our study. We would like to run the simulation for 50 years, and see what is happening every year.

Note: It is extremely important, when creating models, to make sure the units for each component are consistent. In order to establish the proper habit, unit specification will be **REQUIRED** when defining each model component value or formula.

Constructing the Model

1. Open STELLA by double-clicking on the STELLA icon.
2. STELLA has several layers. For this tutorial we need to be in the Modeling layer. When you open the application you will automatically be in the Map layer. Click on the Model tab on the left side of the STELLA window. (See the sample window in number 3.)
3. You should now have a blank area, a window, labeled "STELLA." Note each of the icons and the names for each icon. A sample window is shown here.



4. Select the Stock icon (first icon from the left in the toolbar) by clicking on it. Release the click.
5. Slide the arrow onto the window *without* holding down the button.
6. Position the Stock in the center of the screen, about 1 inch below the top border of the window. Then click once to deposit it.

7. With the Stock selected, type *Population*. If you make a mistake, backspace to correct it. Press Return.

8. Select the Flow icon (located to the right of the Stock icon on the toolbar) by clicking once on it. Be sure to release your click after you select the icon. Move the cursor onto the screen. Position it about two inches to the left of *Population*, on a horizontal line that slices the Stock in half.

9. Click and hold the mouse button. Then, drag the cursor to the right until it makes contact with the left wall of the Stock (the Stock should turn blue on contact). Release the click.

10. While the Flow is selected (dark), name it *Births*. Press Return.

11. Select the Converter icon (third icon from the left on the toolbar). Deposit it about an inch or so below and about 2 inches left of *Population*. Name it: *Birth Fraction*.

12. Select the Connector icon (it looks like an arrow with a circle at the tail). Place the cursor *inside* the converter for *Birth Fraction*. Click and hold. Drag the cursor out of the Converter until it makes contact with the circular part of the *Births* flow (the latter will turn blue). Release your click.

13. Select the Connector icon again. Place the connector cursor inside the *Population* stock. Click and hold. Drag the cursor out of the Stock and make contact with the circular part of the *Births* flow (the latter will turn blue). Release your click.

14. If you haven't done so already select Save from the File menu. Give your model a name and then click on Save.

15. Use the Arrow to double click on the circular part of the *Births* flow. A new window should appear. This window will allow you to define how *Births* are to be calculated during the simulation.

16. Click once on *Population* under "Required Inputs." Then use the calculator to type the * key. Then click on *Birth Fraction*. You should have the equation:

$$\textit{Births} = \textit{Population} * \textit{Birth Fraction}$$

In order to specify the units (for births) in the same section as the formula you must use the curly braces {} to enclose the units. Place the following unit designation {people/year} after the words *Birth Fraction* in the previous definition of births that you just finished. The formula should now be:

$$\textit{Births} = \textit{Population} * \textit{Birth Fraction} \text{ \{people/year\}}$$

Click on OK when done.

Question: What does the equation mean (in your own words)? Do not just repeat the equation. Explain what it means in the population we are studying. I'll get you started: The equation means that we can figure out how many children are born each year by determining...

What assumption is being made here about the growth of the population over time? In other words, is the birth fraction changing over time? Why or why not? (*Hint:* Look back on the first two pages of this tutorial for the word "assume.")

17. With the Arrow, double click on *Population*.

18. Type in the number 1e5 or 100000 {people}. Click on OK when you're finished.

Question: What does this number represent?

19. With the Arrow, double click on *Birth Fraction*. Define it to be 15/1000 {people/people/year} or just {1/year}. Click on OK when done.

Question: What does the value "15/1000" represent (in your own words)?

Setting up the Simulation and Graph

20. Choose Run Specs... from the run menu.

21. Keep the "From" time as 0. Hit the Tab key until you highlight the "To" time box. Type in 50 for the stop time. Then press Tab again and change the DT to 1.00. Click "Years." Click OK.

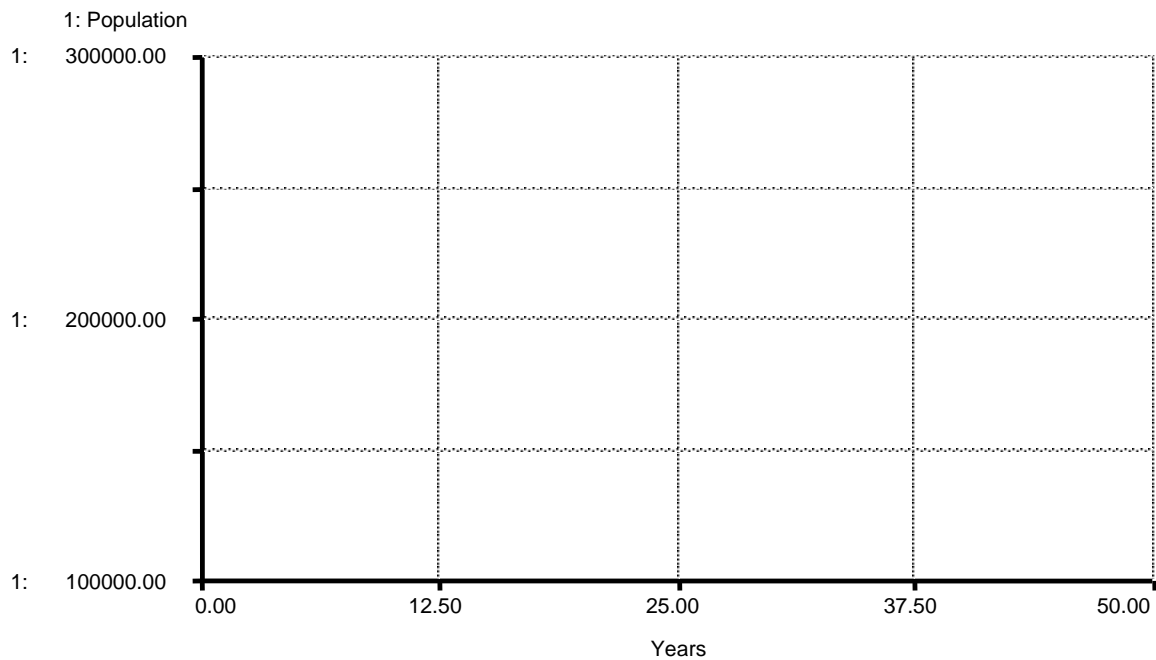
Question: What is the purpose of changing these numbers? What do these numbers mean? (Explain the 0, 50, and the 1.)

22. Click on the Graph icon in the toolbar. Place the icon two inches below and to the right of the *Population* stock. Click once to deposit the graph icon and it will automatically open. Once the graph opens double click on it.

23. Click on *Population* in the "Allowable" window. Then click on the >> button to place *Population* in the "Selected" window. Click OK.

24. Select Save from the file menu to save all the work you've done so far.

25. Sketch a graph of the reference behavior for the population values on the blank grid below. The reference behavior is the behavior that you think will happen to the population over time. (Make the sketch with a dotted curve, rather than a solid curve.)



Run the simulation by selecting the run menu sliding down to Run in that menu. Sketch the graph (using a solid curve) that STELLA produced on the same grid (above) in which you drew your prediction.

How did your prediction compare to the graph produced by the computer?

Part II: The Death Flow and Balancing Feedback Loop

It does not make sense that a population would only increase. We know that deaths occur each year, causing a total population value to remain more stable. As our first experiment, and to keep the model very simple, let us assume that deaths occur from old age only. Let's also assume that the average life span of the population is 76 years (for both men and women). We will add a component to our population that is labeled "Lifespan." We will assume that the value we will assign will remain constant over the entire simulation. This is a major assumption, but not an unreasonable place to start in the study of our first population. The number of people who die each year will be determined by calculating $1/76$ of our total population. In other words, the people in the population reside in the stock for an average of 76 years. So $1/76$ of the total population will flow out (as deaths) each year of the simulation.

We know that deaths have the effect of reducing the number of people in a population (deaths are determined by taking away a percentage of the total population). If, for some reason, the population (stock value) became larger, the number of deaths (outflow) would become larger. An increase in the number of deaths would drive the population (stock value) downward. The effect of this cycle is to bring the population into balance. This is referred to as "balancing" feedback.

Constructing More of the Model

26. Select the Flow icon again. Position the flow cursor *inside* the *Population* Stock. Click and hold. Then, drag the icon out of the Stock (through the right wall) to a distance about two inches from the wall. Release the click.

27. Name the outflow *Deaths*. Click anywhere on the blank screen to deselect *Deaths*.

28. Select the Converter icon again. Deposit a converter below, and three inches to the right of *Population*. Name it: *Lifespan*.

29. Select the Connector icon. Use it to link *Population* to *Deaths*.

30. By depressing the "option" (Mac) or "alt" (Win) key you can recall the previously selected tool. Do that to recall the Connector icon. Place the connector cursor *inside* the Converter for *Lifespan*. Click and hold. Use it to link *Lifespan* to *Deaths*.

31. With the Arrow, double click on *Deaths*. Define *Deaths* as the product of *Population* and $1/Lifespan$.

Question: What does the *Deaths* equation look like? (Write it below.)

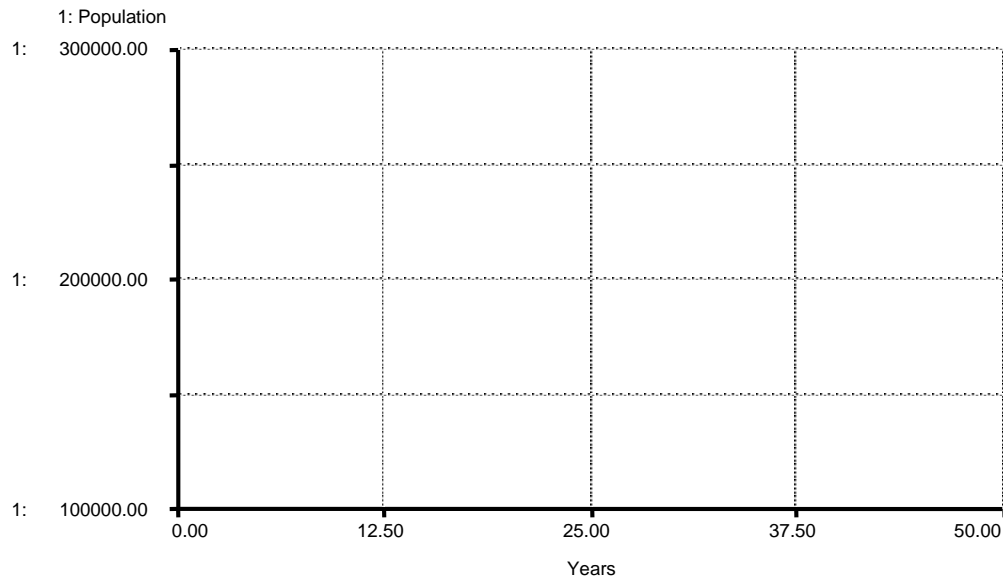
Did you remember to include the units? Click OK when done.

32. With the Arrow, double-click on *Lifespan*. Define it to be 76 {years}. Click on OK.

Question: What does the value "76" represent (in your own words)?

33. Now would be a good time to Save your model again.

34. Sketch a graph of the reference behavior for *Population* on the blank grid below. (Make the sketch with a dotted curve, rather than a solid curve).



Run the simulation by selecting Run from the main menu bar and sliding down to Run in that menu. Sketch the graph (using a solid curve) that STELLA produced on the same grid (above) in which you drew your prediction. (Be careful. STELLA may have changed the vertical scale for its graph. You will learn how to control this a little later.)

Describe the difference between the model output and your reference behavior pattern.

Creating a Table for Numerical Output

35. Click once on the Table icon in the toolbar. (It is located to the right of the graph icon.) Deposit the table icon below the "Graph 1" icon by clicking once. Once the table opens double click on it.

36. Enter *Population* into the table by double clicking on it in the Allowable Window. It should have moved to the Selected Window. Click OK.

37. Choose Run from the run menu. Record the population value at 50 years (here and on another piece of paper to use for the next assignment).

(*Note:* You may resize the width of each column in the table by pointing the cursor to the line separating two columns of numbers. Click and drag the line to the right or left. Release your click.)

38. Save your work to your disk. (*Note:* All diagrams, graphs, and tables are saved as part of your STELLA file.)

Part III: Sensitivity Analysis (Testing Multiple Values)

This feature will allow us to see more than one population graph at a time. We will alter the birth fraction five times to see how that affects the growth of the population.

39. Choose Sensi Specs... from the Run menu.

40. Click on *Birth Fraction* in the Allowable window. Click the >> button.

41. Change the number of runs to 5.

42. Click on *Birth Fraction* in the Selected window.

43. Click on the Ad hoc button. Do you know what ad hoc means? Find out and give a definition here.

44. Type in .01. Click on the Set button. Type in .02. Click on the Set button. Type in .03, .045, and .0545 following the same procedure. How many children are being born (per 1000 population) each year? (Fill out the table below.)

Birth Fraction (as a decimal)	Number of Newborns per 1000 population
.01	
.02	
.03	
.045	
.0545	

45. Click on the Graph button located in the lower left of the window. You should now be looking at the Define Graph window.

46. Click on *Population* in the small Allowable window in the upper left of the screen. Click on the >> button.

47. Click on *Population* in the small Selected window. Click on the double-headed arrow to the right of *Population*. (If the double-headed arrow does not have a bar above and below it you may need to click on the arrow again. *Note:* This process forces the vertical scale to be set to the values chosen by the modeler.)

48. Click in the Min box. Type 0. Press Tab. Type 800000 in the Max box. Click on the Set button. Click OK.

49. Choose S-Run from the run menu. After all the graphs are drawn, double-click on the graph pad again. In the title section indicate that graph 1 = .01 birth fraction, 2 = .02, 3 = .03, 4 = .045, 5 = .0545. It is important to include this information on the graph pad so the reader knows why the graphs are different. Print out the graph.

50. What do you notice about the population growth shown by each of the five population graphs? (Relate the appearance of the graph to the specific number of newborns per 1000 population.)

51. For this question you must do research and *indicate where you found your answers*. Use numbers wherever possible to support your answers. Sources for information must be acceptable, so ask the teacher if you are unsure of the acceptability of the source. (Databases of world statistics and encyclopedias are acceptable sources.)

Name at least 2 countries that tend to have a low birth fraction. Give at least two good reasons (for each country) why the birth fraction would be low. Use characteristics about the countries you identified to support why the birth fraction is low for those specific countries.

Name at least 2 countries that tend to have a high birth fraction. Give at least two good reasons (for each country) why the birth fraction would be high. Use characteristics about the countries you identified to support why the birth fraction is high for those specific countries.

52. What sort of problems occur (name at least 2) when a country sustains a high birth fraction for several generations? (No research is required.)



Answer question J1 in your journal. Then pick either question J2 or J3 to also answer in your journal.

J1. What is the current United States birth fraction? Death fraction? Is the trend up or down? Is it different in different parts of the country? If it is different, why do you think there is a difference?

J2. What are the implications of China's policy on one birth per family. What effect might this have on future generations in China?

J3. Find an article or graph from the news dealing with population trends. Explain why the reporter thought the public should be concerned or interested in the trend. Paste the article in your journal. Make a note of the source and date for the article.